

LEARNING

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CONTENTS

Take vertices $0, 1, \dots, n$ with indices modulo $n + 1$. Add arrows in both directions between adjacent vertices:

$$a_i : i \rightarrow i + 1, \quad a_i^* : i + 1 \rightarrow i$$

So the quiver looks like a cycle with forward and backward arrows everywhere.

Form the path algebra we get the following relation at each vertex i :

$$a_i a_i^* - a_{i-1}^* a_{i-1} = 0$$

or

$$a_i a_i^* = a_{i-1}^* a_{i-1}$$

Hence we obtain that

$$a_0 a_0^*, a_1 a_1^*, \dots, a_n a_n^*$$

represent the same element (up to base point). Call this element z .

We now define three closed paths starting and ending at vertex 0.

$$x = a_0 a_1 \cdots a_n$$

$$y = a_n^* a_{n-1}^* \cdots a_0^*$$

$$z = a_0 a_0^*$$

(which equals every $a_i a_i^*$ by the relation above). Consider

$$xy = (a_0 a_1 \cdots a_n) (a_n^* a_{n-1}^* \cdots a_0^*)$$

In the middle we get

$$xy = a_0 a_1 \cdots a_{n-1} (a_n a_n^*) a_{n-1}^* \cdots a_0^*$$

Using the relation

$$a_n a_n^* = a_{n-1}^* a_{n-1}$$

we replace

$$xy = a_0 a_1 \cdots a_{n-1} (a_{n-1}^* a_{n-1}) a_{n-1}^* \cdots a_0^*$$

Repeat the same again

$$a_{n-1} a_{n-1}^* = a_{n-2}^* a_{n-2}$$

Continuing inductively, everything collapses until

$$(a_0 a_0^*)^{n+1}$$

So

$$xy = (a_0 a_0^*)^{n+1} = z^{n+1}$$